Fundamentals of Accelerators - 2012

Lecture - Day 6

William A. Barletta
Director, United States Particle Accelerator School
Dept. of Physics, MIT
Faculty of Economics, U. Ljubljana

What sets the discovery potential of colliders?

1. **Energy**
   - → determines the scale of phenomena to be studied
2. **Luminosity** (collision rate)
   - → determines the production rate of "interesting" events

$$\text{Luminosity} = \frac{\text{Energy} \times \text{Current}}{\text{Focal depth} \times \text{Beam quality}}$$

   - → Scale L as E^2 to maximize discovery potential at a given energy
   - → Factor of 2 in energy worth factor of 10 in luminosity

✳ Critical limiting technologies:

- → Energy - Dipole fields, accelerating gradient, machine size
- → Current - Synchrotron radiation, wake fields
- → Focal depth - IR quadrupole gradient
- → Beam quality - Beam source, machine impedance, feedback

Beams have internal (self-forces)

- Space charge forces
  - Like charges repel
  - Like currents attract
- For a long thin beam

$$E_{sp}(V/cm) = \frac{60\ I_{beam}(A)}{R_{beam}(cm)}$$

$$B_{\theta}(gauss) = \frac{I_{beam}(A)}{5\ R_{beam}(cm)}$$

Net force due to transverse self-fields

In vacuum:

Beam's transverse self-force scale as $1/\gamma^2$

→ Space charge repulsion: $E_{sp,\perp} \sim N_{beam}$

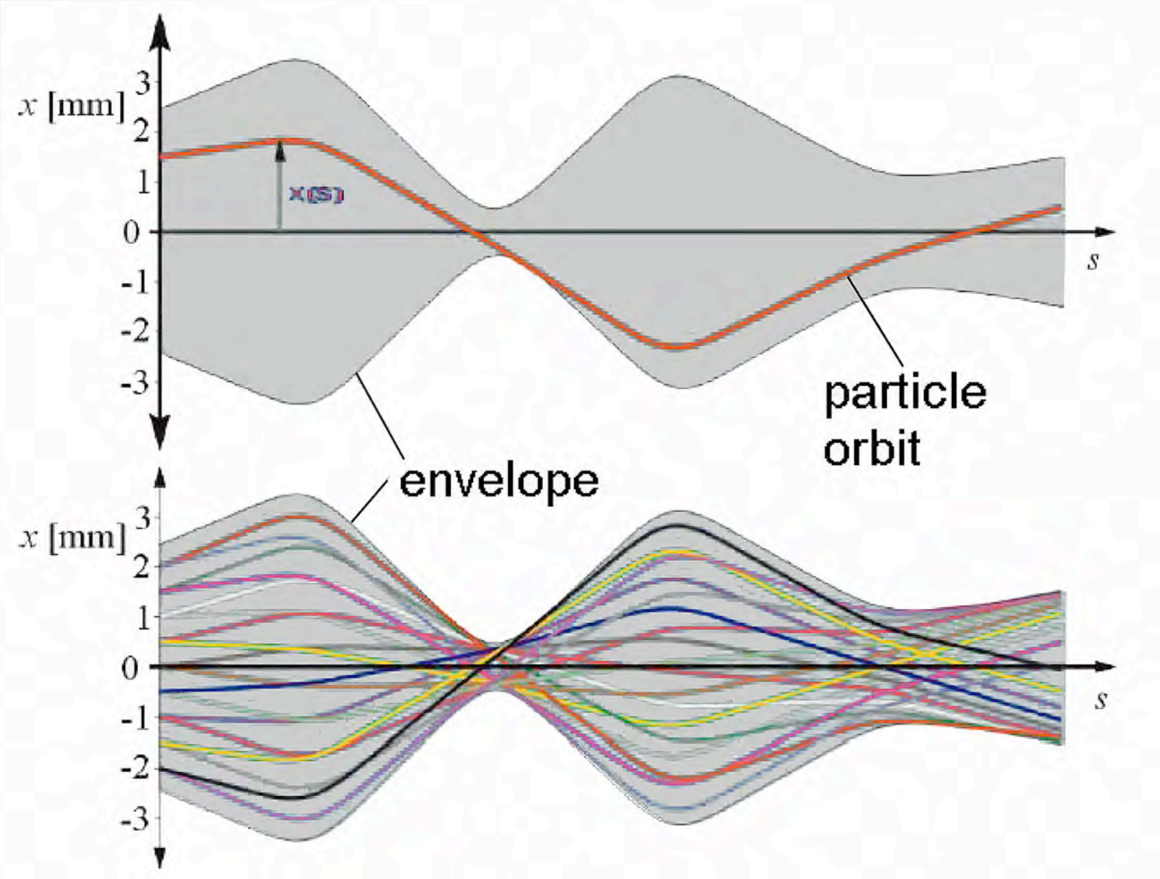
→ Pinch field: $B_\theta \sim I_{beam} \sim v_z N_{beam} \sim v_z E_{sp}$

$$\therefore F_{sp,\perp} = q\,(E_{sp,\perp} + v_z \times B_\theta) \sim (1-v^2)\, N_{beam} \sim N_{beam}/\gamma^2$$

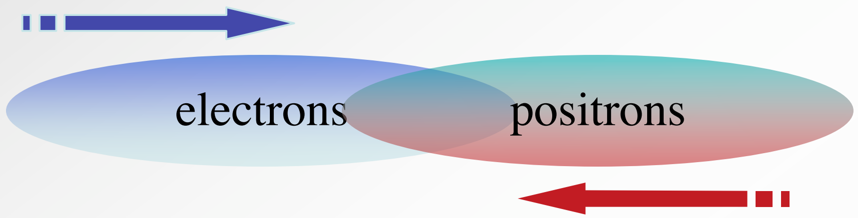
Beams in collision are ***not*** in vacuum (beam-beam effects)

We see that ε characterizes the beam while β(s) characterizes the machine optics

- β(s) sets the physical aperture of the accelerator because the beam size scales as $\sigma_x(s) = \sqrt{\varepsilon_x \beta_x(s)}$



Example: Megagauss fields in linear collider



At Interaction Point space charge cancels; currents add

==> strong beam-beam focus

--> Luminosity enhancement

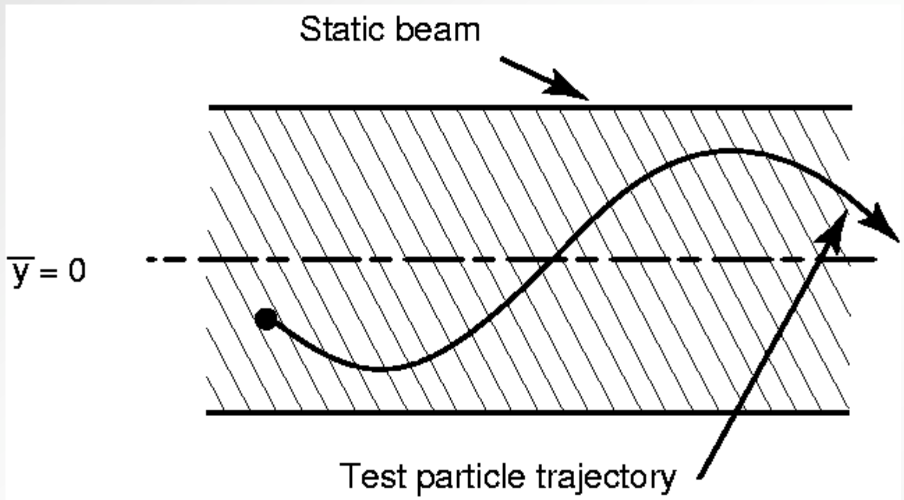
--> Very strong synchrotron radiation

Consider 250 GeV beams with 1 kA focused to 100 nm

$B_{peak} \sim 40$ Mgauss

==> Large $\Delta E/E$

Types of tune shifts: Incoherent motion



- Center of mass does not move
- Beam environment does not "see" any motion
- Each particle is characterized by an individual amplitude & phase

From: E. Wilson Adams lectures

Incoherent collective effects

- Beam-gas scattering
  - → Elastic scattering on nuclei => leave physical aperture
  - → Bremsstrahlung
  - → Elastic scattering on electrons
  - → Inelastic scattering on electrons

  (Bremsstrahlung, elastic and inelastic scattering on electrons: } leave rf-aperture)

  =======> reduce beam lifetime
- Ion trapping (also electron cloud) - scenario
  - → Beam losses + synchrotron radiation => gas in vacuum chamber
  - → Beam ionizes gas
  - → Beam fields trap ions
  - → Pressure increases linearly with time
  - → Beam -gas scattering increases
- Intra-beam scattering

Intensity dependent effects

- Types of effects
  - Space charge forces in individual beams
  - Wakefield effects
  - Beam-beam effects

- General approach: solve

$$x'' + K(s)x = \frac{1}{\gamma m \beta^2 c^2} F_{non-linear}$$

- For example, a Gaussian beam has

$$F_{SC} = \frac{e^2 N}{2\pi\varepsilon_o \gamma^2 r}\left(1 - e^{-r^2/2\sigma^2}\right) \quad where\ N = \ charge/unit\ length$$

- For r < σ

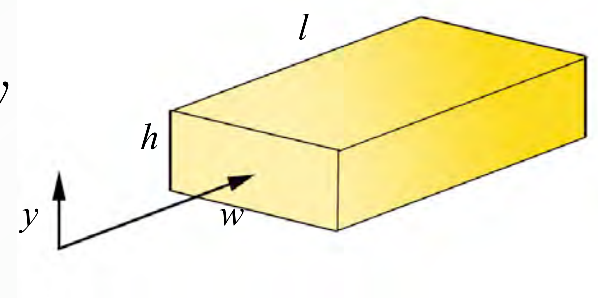
$$F_{SC} \approx \frac{e^2 N}{4\pi\varepsilon_o \gamma^2} r$$

Beam-beam tune shift

$$\Delta p_y(y) \sim \left(F_{elec} + F_{mag}\right)\frac{l}{2c}$$ *(once the particle passes l/2 the other bunch has passed)*

- For $\gamma >> 1,\ F_{elec} \approx F_{mag}$

$$\mathcal{E} = \frac{N_{beam} e}{lwh} y$$



$$\therefore \Delta p_y(y) \approx 2\frac{e\mathcal{E}}{2c} = \frac{e^2 N_B}{\varepsilon_o cwh} y$$

$$\Rightarrow \frac{\Delta p_y}{p_o} = \Delta y' \sim y$$ similar to gradient error $k_y \Delta s$ with $k_y \Delta s = \frac{\Delta y'}{y}$

- Therefore the tune shift is

$$\Delta Q = -\frac{\beta^*}{4\pi} k_y \Delta s \approx \frac{r_e \beta^* N}{\gamma wh}$$ where $r_e = \frac{e^2}{4\pi\varepsilon_o mc^2}$

- For a Gaussian beam

$$\Delta Q \approx \frac{r_e}{2}\frac{\beta^* N}{\gamma A_{\text{int}}}$$

Effect of tune shift on luminosity

- The luminosity is $$\mathcal{L} = \frac{f_{coll} N_1 N_2}{4 A_{\text{int}}}$$
- Write the area in terms of emittance & β at the IR
$$A_{\text{int}} = \sigma_x \sigma_y = \sqrt{\beta_x^* \varepsilon_x} \circ \sqrt{\beta_y^* \varepsilon_y}$$
- For simplicity assume that
$$\frac{\beta_x^*}{\beta_y^*} = \frac{\varepsilon_x}{\varepsilon_y} \Rightarrow \beta_x^* = \frac{\varepsilon_x}{\varepsilon_y} \beta_y^* \Rightarrow \beta_x^* \varepsilon_x = \frac{\varepsilon_x^2}{\varepsilon_y} \beta_y^*$$
- In that case
$$A_{\text{int}} = \varepsilon_x \beta_y^*$$
- And
$$\mathcal{L} = \frac{f_{coll} N_1 N_2}{4 \varepsilon_x \beta_y^*} \sim \frac{I_{beam}^2}{\varepsilon_x \beta_y^*}$$

Increase N to the tune shift limit

- We saw that

$$\Delta Q_y \approx \frac{r_e}{2} \frac{\beta^* N}{\gamma A_{\text{int}}}$$

or

$$N = \Delta Q_y \frac{2\gamma A_{\text{int}}}{r_e \beta^*} = \Delta Q_y \frac{2\gamma \varepsilon_x \beta^*}{r_e \beta^*} = \frac{2}{r_e} \gamma \varepsilon_x \Delta Q_y$$

Therefore the tune shift limited luminosity is

$$\mathcal{L} = \frac{2}{r_e} \Delta Q_y \frac{f_{coll} N_1 \gamma \varepsilon_x}{4 \varepsilon_x \beta_y^*} \sim \Delta Q_y \left(\frac{IE}{\beta_y^*} \right)$$

Incoherent tune shift for in a synchrotron

Assume: 1) an unbunched beam (no acceleration), & 2) uniform density in a circular x-y cross section (not very realistic)

$$x'' + \left(K(s) + \underline{K_{SC}(s)}\right)x = 0 \quad \Rightarrow Q_{x0} \text{ (external)} + \Delta Q_x \text{ (space charge)}$$

For small "gradient errors" k_x

$$\underline{\Delta Q_x} = \frac{1}{4\pi}\int_0^{2R\pi} k_x(s)\beta_x(s)ds = \frac{1}{4\pi}\int_0^{2R\pi} \underline{K_{SC}}(s)\beta_x(s)ds$$

where

$$K_{SC} = -\frac{2r_0 I}{ea^2\beta^3\gamma^3 c}$$

$$\Delta Q_x = -\frac{1}{4\pi}\int_0^{2\pi R}\frac{2r_0 I}{e\beta^3\gamma^3 c}\frac{\beta_x(s)}{a^2}ds = -\frac{r_0 RI}{e\beta^3\gamma^3 c}\left\langle\frac{\beta_x(s)}{a^2(s)}\right\rangle = -\frac{r_0 RI}{e\beta^3\gamma^3 c\varepsilon_x}$$

From: E. Wilson Adams lectures

Incoherent tune shift limits current *at injection*

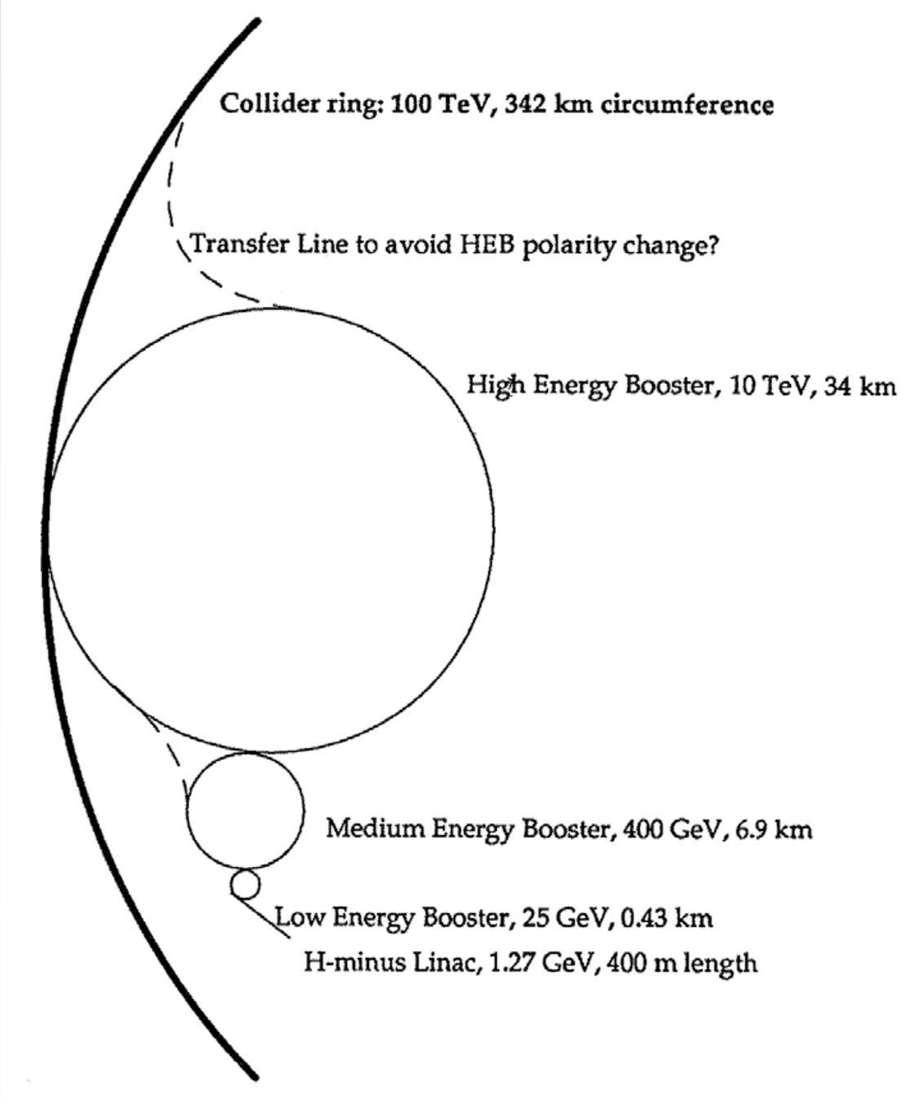
$$\Delta Q_{x,y} = -\frac{r_0}{2\pi\beta^2\gamma^3}\frac{N}{\varepsilon_{x,y}}$$

using I = (Neβc)/(2πR) with
N…number of particles in ring
$\varepsilon_{x,y}$….emittance containing 100% of particles

- "Direct" space charge, unbunched beam in a synchrotron
- Vanishes for γ » 1
- Important for low-energy hadron machines
- *Independent of machine size* 2πR for a *given N*
- *Overcome by higher energy injection ==> cost*

From: E. Wilson Adams lectures

Injection chain for a 200 TeV Collider



Beam lifetime

Based on F. Sannibale USPAS Lecture

Finite aperture of accelerator ==> loss of beam particles

- Many processes can excite particles on orbits larger than the nominal.
  - ➙ If new orbit displacement exceed the aperture, the particle is lost
- The limiting aperture in accelerators can be either *physical* or *dynamic*.
  - ➙ Vacuum chamber defines the physical aperture
  - ➙ Momentum acceptance defines the dynamical aperture

From: Sannibale USPAS Lecture

Important processes in particle loss

- Gas scattering, scattering with the other particles in the beam, quantum lifetime, tune resonances, &collisions
- Radiation damping plays a major role for electron/positron rings
  - ➙For ions, lifetime is usually much longer
    - Perturbations progressively build-up & generate losses
- Most applications require storing the beam as long as possible

==> limiting the effects of the residual gas scattering

==> ultra high vacuum technology

What do we mean by lifetime?

- Number of particles lost at time *t* is proportional to the number of particles present in the beam at time *t*

$$dN = -\alpha N(t)\,dt \qquad with \quad \alpha \equiv constant$$

- Define the lifetime τ = 1/α; then $N = N_0 e^{-t/\tau}$

- Lifetime is the time to reduce the number of beam particles to 1/*e* of the initial value

- Calculate the lifetime due to the individual effects (gas, Touschek, …)

$$\frac{1}{\tau_{total}} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + ...$$

From: Sannibale USPAS Lecture

Is the lifetime really constant?

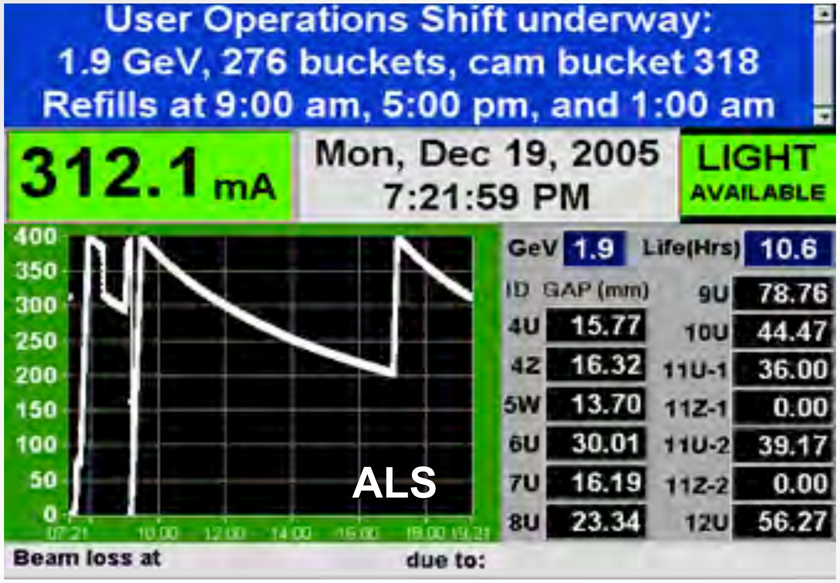
- In typical electron storage rings, lifetime depends on beam current
- Example: the *Touschek effect* losses depend on current.
  - ➞ When the stored current decreases, the losses due to Touschek decrease ==> lifetime increases
- Example: Synchrotron radiation radiated by the beam desorbs gas molecules trapped in the vacuum chamber
  - ➞ The higher the stored current, the higher the synchrotron radiation intensity and the higher the desorption from the wall.
  - ➞ Pressure in the vacuum chamber increases with current

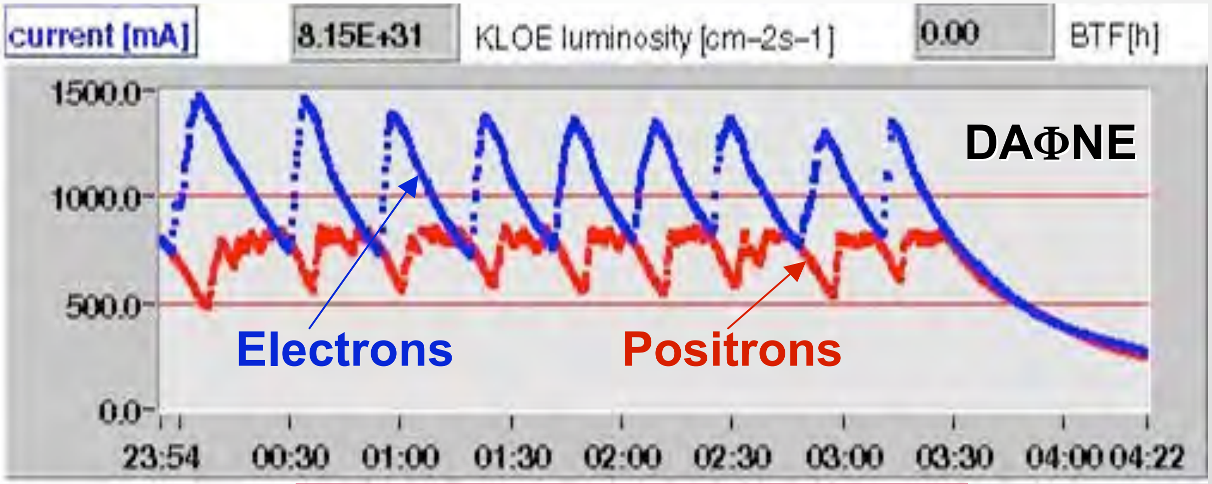
  ==> increased scattering between the beam and the residual gas

  ==> reduction of the beam lifetime

From: Sannibale USPAS Lecture

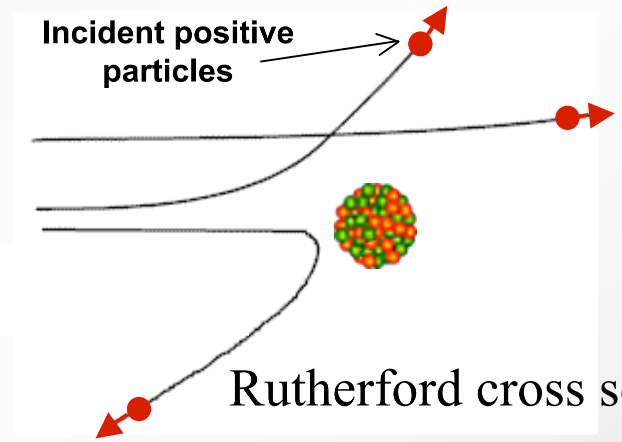
Examples of beam lifetime measurements





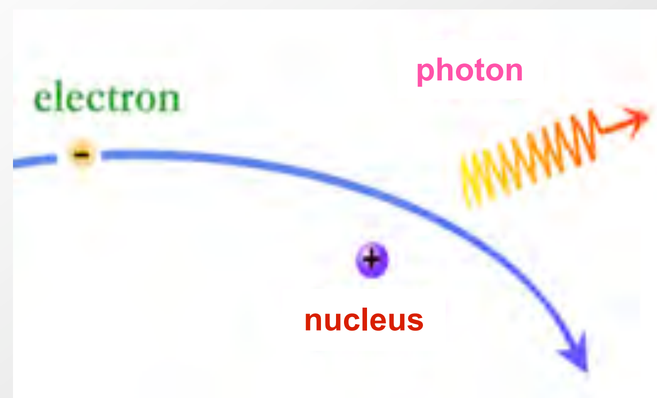
Beam loss by scattering

- Elastic (Coulomb scattering) from residual background gas
  - Scattered beam particle undergoes transverse (betatron) oscillations.
  - If the oscillation amplitude exceeds ring acceptance the particle is lost



Rutherford cross section

- Inelastic scattering causes particles to *lose energy*
  - Bremsstrahlung or atomic excitation
  - If energy loss exceeds the momentum acceptance the particle is lost



Bremsstrahlung cross section

Elastic scattering loss process

- Loss rate is $$\left.\frac{dN}{dt}\right|_{Gas} = -\phi_{beam\ particles} N_{molecules} \sigma^*_R$$

$$\phi_{beam\ particles} = \frac{N}{A_{beam} T_{rev}} = \frac{N}{A_{beam}} \frac{\beta c}{L_{ring}}$$

$$N_{molecules} = n A_{beam} L_{ring}$$

$$\sigma^*_R = \int_{Lost} \frac{d\sigma_{Rutherford}}{d\Omega} d\Omega = \int_0^{2\pi} d\varphi \int_{\theta_{MAX}}^{\pi} \frac{d\sigma_{Rutherford}}{d\Omega} \sin\theta \, d\theta$$

$$\frac{d\sigma_R}{d\Omega} = \frac{1}{(4\pi\varepsilon_0)^2} \left(\frac{Z_{beam} Z_{gas} e^2}{2\beta c\, p} \right)^2 \frac{1}{\sin^4(\theta/2)} \quad [MKS]$$

Gas scattering lifetime

- Integrating yields

$$\left.\frac{dN}{dt}\right|_{Gas} = -\frac{\pi\, n\, N\beta\, c}{(4\pi\,\varepsilon_0)^2}\left(\frac{Z_{Inc} Ze^2}{\beta\, c\, p}\right)^2 \frac{1}{\tan^2(\theta_{MAX}/2)}$$

Loss rate for gas elastic scattering [*MKS*]

- For M-atomic molecules of gas $n = M\, n_0 \dfrac{P_{[Torr]}}{760}$

- For a ring with acceptance ε_A & for small θ $\langle\theta_{MAX}\rangle = \sqrt{\dfrac{\varepsilon_A}{\langle\beta_n\rangle}}$

==>

$$\tau_{Gas} \cong \frac{760}{P_{[Torr]}} \frac{4\pi\varepsilon_0^2}{\beta\, c\, M\, n_0}\left(\frac{\beta\, c\, p}{Z_{Inc} Ze^2}\right)^2 \frac{\varepsilon_A}{\langle\beta_T\rangle} \quad [MKS]$$

Inelastic scattering lifetimes

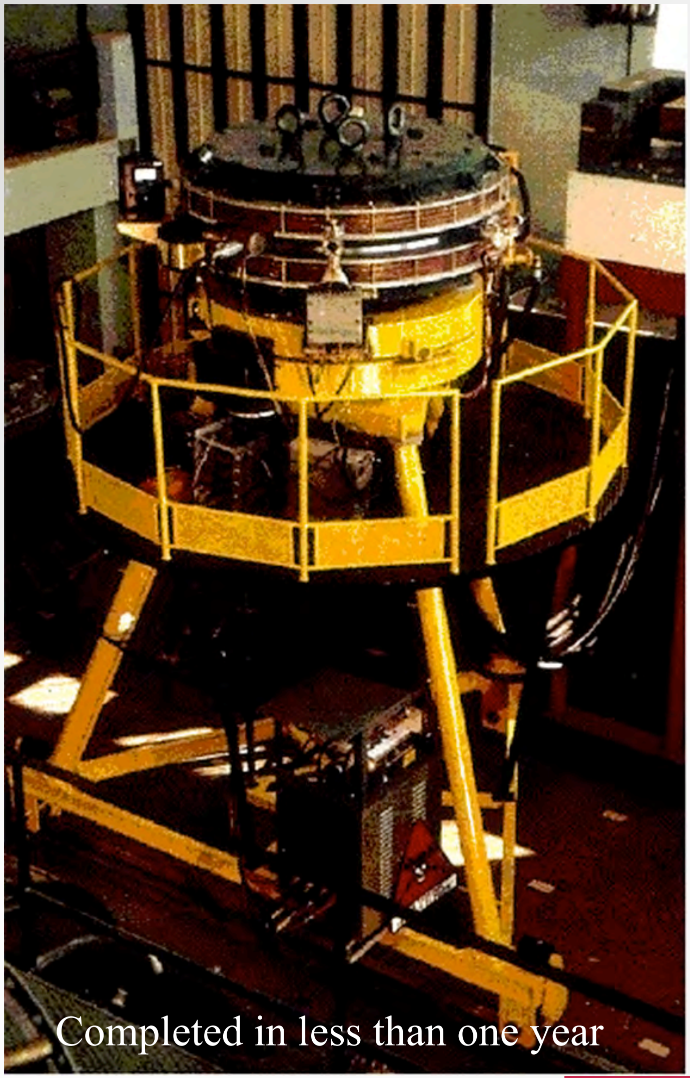
- Beam-gas bremsstrahlung: if E_A is the energy acceptance

$$\tau_{Brem[hours]} \cong -\frac{153.14}{\ln\left(\Delta E_A / E_0\right)} \frac{1}{P_{[nTorr]}}$$

- Inelastic excitation: For an average β_n

$$\tau_{Gas[hours]} \cong 10.25 \frac{E^2_{0[GeV]}}{P_{[nTorr]}} \frac{\varepsilon_{A[\mu m]}}{\langle \beta_n \rangle_{[m]}}$$

ADA - The first storage ring collider (e^+e^-) by B. Touschek at Frascati (1960)



Completed in less than one year

The storage ring collider idea was invented by R. Wiederoe in 1943

- Collaboration with B. Touschek
- Patent disclosure 1949

Erteilt auf Grund des Ersten Überleitungsgesetzes vom 8. Juli 1949
(WiGBl. S. 175)

BUNDESREPUBLIK DEUTSCHLAND

AUSGEGEBEN AM
11. MAI 1953

DEUTSCHES PATENTAMT

PATENTSCHRIFT

№ 876 279

KLASSE 21g GRUPPE 36

W 687 VIIIc / 21g

Dr.-Ing. Rolf Wideröe, Oslo
ist als Erfinder genannt worden

Aktiengesellschaft Brown, Boveri & Cie, Baden (Schweiz)

Anordnung zur Herbeiführung von Kernreaktionen

Patentiert im Gebiet der Bundesrepublik Deutschland vom 8. September 1949 an
Patentanmeldung bekanntgemacht am 18. September 1952
Patenterteilung bekanntgemacht am 26. März 1953

Kernreaktionen können dadurch herbeigeführt werden, daß geladene Teilchen von hoher Geschwindigkeit und Energie, in Elektronenvolt gemessen, auf die zu untersuchenden Kerne geschossen werden. Wenn die geladenen Teilchen in einen gewissen Mindestabstand von den Kernen gelangen, werden die Kernreaktionen eingeleitet. Da aber neben den zu untersuchenden Kernen noch die gesamten Elektronen der Atomhülle vorhanden sind und auch der Wirkungsquerschnitt des Kernes sehr klein ist, wird der größte Teil der geladenen Teilchen von den Hüllenelektronen abgebremst, während nur ein sehr kleiner Teil die gewünschten Kernreaktionen herbeiführt.

Erfindungsgemäß wird der Wirkungsgrad der Kernreaktionen dadurch wesentlich erhöht, daß die Reaktion in einem Vakuumgefäß (Reaktionsröhre) durchgeführt wird, in welchem die geladenen Teilchen hoher Geschwindigkeit gegen einen Strahl von den zu untersuchenden und sich entgegengesetzt bewegenden Kernen auf einer sehr langen Strecke laufen müssen. Dies kann in der Weise durchgeführt werden, daß die geladenen Teilchen zum mehrmaligen Umlauf in einer Kreisröhre gezwungen werden, wobei die zu untersuchenden Kerne auf derselben Kreisbahn, aber in entgegengesetzter Richtung umlaufen. Da die geladenen Teilchen dabei nicht von bei der Reaktion unwirksamen Elektronen abgebremst werden und andererseits auf einer sehr langen Wegstrecke gegen die Kerne sich bewegen können, wird die Wahrscheinlichkeit für das Eintreten der Kernreaktionen wesentlich größer und der Wirkungsgrad der Reaktion sehr stark erhöht.

Um die bei der Kreisbewegung entstehenden Zentrifugalkräfte aufzuheben, müssen die umlaufenden Teilchen von nach innen gerichteten Ablenkkräften gesteuert werden, während eine Diffusion der Teile mittels stabilisierender, von allen Seiten auf den Bahnkreis gerichteter Kräfte verhindert wird. Falls die gegen-

Touschek effect:
Intra-beam Coulomb scattering

- Coulomb scattering between beam particles can transfer transverse momentum to the longitudinal plane
  - If the $p_{||}+\Delta p_{||}$ of the scattered particles is outside the momentum acceptance, the particles are lost
  - First observation by Bruno Touschek at ADA e^+e^- ring
- Computation is best done in the beam frame where the relative motion of the particles is non-relativistic
  - Then boost the result to the lab frame

$$\frac{1}{\tau_{Tousch.}} \propto \frac{1}{\gamma^3} \frac{N_{beam}}{\sigma_x \sigma_y \sigma_S} \frac{1}{\left(\Delta p_A / p_0\right)^2} \propto \frac{1}{\gamma^3} \frac{N_{beam}}{A_{beam} \sigma_S} \frac{1}{\hat{V}_{RF}}$$

Transverse quantum lifetime

- At a fixed s, transverse particle motion is purely sinusoidal

$$x_T = a\sqrt{\beta_n}\sin\left(\omega_{\beta_n} t + \varphi\right) \qquad T = x, y$$

- Tunes are chosen in order to avoid resonances.
  - ➙ At a fixed azimuthal position, a particle turn after turn sweeps all possible positions between the envelope
- Photon emission randomly changes the "invariant" *a* & consequently changes the trajectory envelope as well.
- Cumulative photon emission can bring the particle envelope beyond acceptance in some azimuthal point
  - ➙ The particle is lost

Quantum lifetime was first estimated by Bruck & Sands

$$\tau_{Q_T} \cong \tau_{D_T} \frac{\sigma_T^2}{A_T^2} \exp\left(A_T^2 / 2\sigma_T^2\right) \quad T = x, y$$

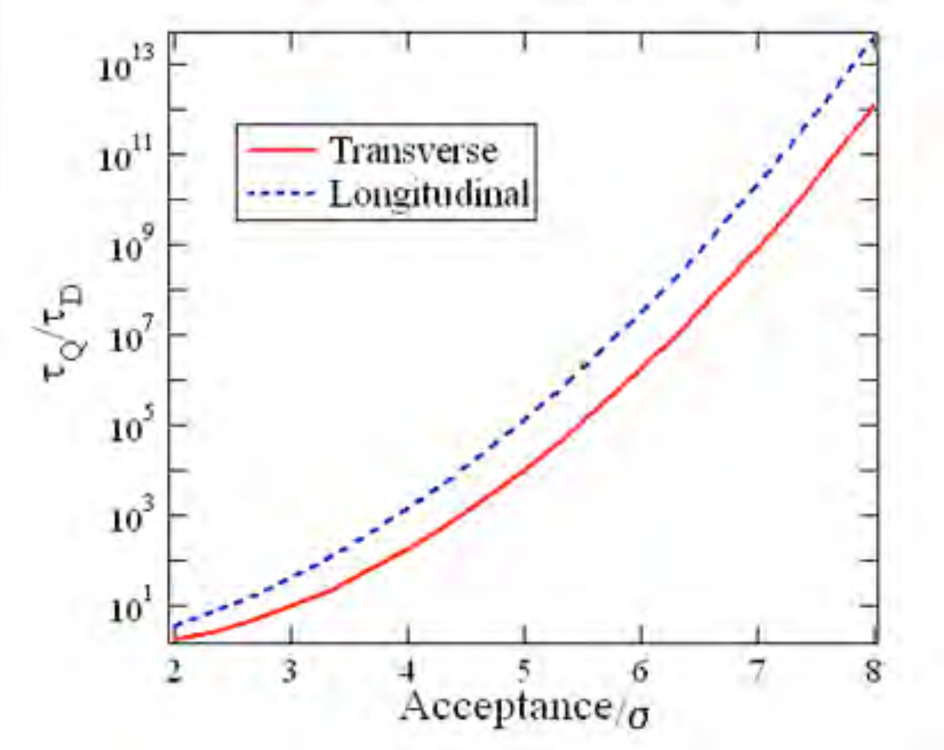
Transverse quantum lifetime

$$where \quad \sigma_T^2 = \beta_T \varepsilon_T + \left(D_T \frac{\sigma_E}{E_0}\right)^2 \quad T = x, y$$

$\tau_{D_T} \equiv$ *transverse damping time*

$$\tau_{Q_L} \cong \tau_{D_L} \exp\left(\Delta E_A^2 / 2\sigma_E^2\right)$$

Longitudinal quantum lifetime



- Quantum lifetime varies very strongly with the ratio between acceptance & rms size.

Values for this ratio ≥6 are usually required

Lifetime summary

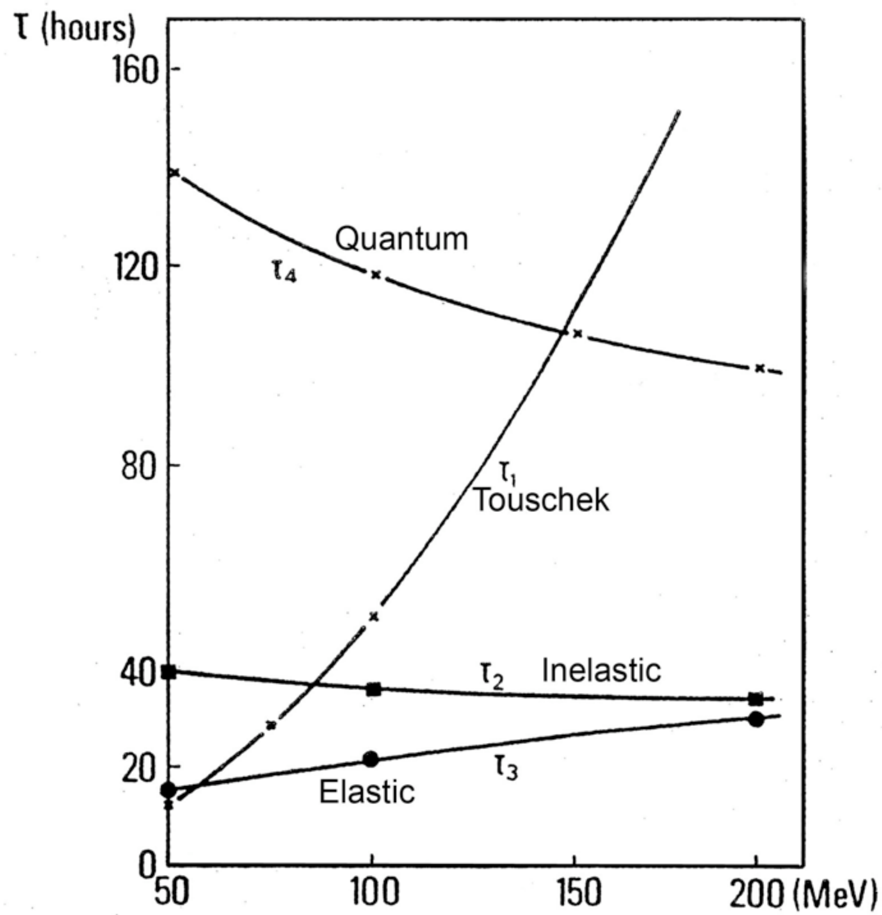


Fig. 1. Lifetime resulting from the different types of beam–gas interaction

In colliders the beam-beam collisions also deplete the beams

This gives the luminosity lifetime

LEP-3

Life gets hard very fast

William A. Barletta
Director, US Particle Accelerator School
Dept. of Physics, MIT
Economics Faculty, University of Ljubljana

Physics of a Higgs Factory

- Dominant decay reaction is $e^+ + e^- \Rightarrow H \Rightarrow W + Z$
- $M_W + M_Z = 125 + 91.2$ GeV/c^2

  ==> set our CM energy a little higher: **~240 GeV**
- Higgs production cross section ~ 220 fb (2.2 x 10^{-37} cm^2)
- Peak $\mathcal{L} = 10^{34}$ cm^{-1} s^{-1} = $< \mathcal{L} > \sim 10^{33}$ cm^{-1} s^{-1}
- ~30 fb^{-1} / year ==> 6600 Higgs / year
- Total cross-section at ~ 100 pb•(100GeV/E)^2

We don't have any choice about these numbers

Tune shift limited luminosity of the collider

$$L = \frac{N^2 c\gamma}{4\pi\varepsilon_n \beta^* S_B} = \frac{1}{e r_i m_i c^2} \frac{N r_i}{4\pi\varepsilon_n} \left(\frac{EI}{\beta^*}\right) = \frac{1}{e r_i m_i c^2} \frac{N r_i}{4\pi\varepsilon_n} \left(\frac{P_{beam}}{\beta^*}\right) \qquad i = e, p$$

Linear or Circular

Tune shift

Or in practical units for electrons

$$L = 2.17 \circ 10^{34} \left(1 + \frac{\sigma_x}{\sigma_y}\right) Q_y \left(\frac{1\ \text{cm}}{\beta^*}\right) \left(\frac{E}{1\ \text{GeV}}\right) \left(\frac{I}{1\ \text{A}}\right)$$

At the tune shift limit $\left(1 + \frac{\sigma_x}{\sigma_y}\right) Q_y \approx 0.1$

$$L = 2.17 \circ 10^{33} \left(\frac{1\ \text{cm}}{\beta^*}\right) \left(\frac{E}{1\ \text{GeV}}\right) \left(\frac{I}{1\ \text{A}}\right)$$

We can only choose I(A) and ß*(cm)

- For the LHC tunnel with $f_{dipole} = 2/3$, $\rho \sim 3000$ m
- Remember that

$$\rho(\mathrm{m}) = 3.34 \left(\frac{p}{1\ \mathrm{GeV/c}}\right)\left(\frac{1}{q}\right)\left(\frac{1\ \mathrm{T}}{B}\right)$$

- Therefore, $B_{max} = 0.134$ T
- Each beam particle will lose to synchrotron radiation

$$U_o(keV) = 88.46 \frac{E^4(GeV)}{\rho(m)}$$

or 6.2 GeV per turn

$$I_{beam} << 0.1\ A$$

I_{beam} = 7.5 mA ==> ~100 MW of radiation

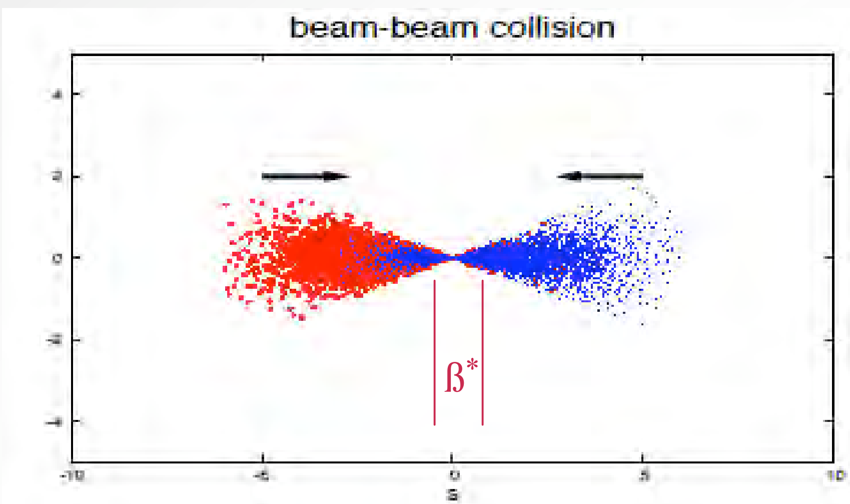
- Then

$$L \approx 2.6 \circ 10^{33} \left(\frac{1\ \mathrm{cm}}{\beta^*} \right)$$

- Therefore to meet the luminosity goal

$$\langle \beta^*_x \beta^*_y \rangle^{1/2} \sim 0.1\ \mathrm{cm}$$

- Is this possible? Recall that is the depth of focus at the IP



The "hourglass effect" lowers $\mathcal{L}$

==> $\beta^* \sim \sigma_z$

Bunch length is determined by V_{rf}

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- The analysis of longitudinal dynamics gives

$$\sigma_S = \frac{c\alpha_C}{\Omega_{sync}} \frac{\sigma_p}{p_0} = \sqrt{\frac{c^3}{2\pi q} \frac{p_0 \beta_0 \eta_C}{h\, f_0^2\, \hat{V} \cos(\varphi_S)}} \frac{\sigma_p}{p_0}$$

  where $\alpha_c = (\Delta L/L) / (\Delta p/p)$ must be ~ 10^{-5} for electrons to remain in the beam pipe

- To know bunch length we need to know $\Delta p/p \sim \Delta E/E$

- For electrons to a good approximation

$$\Delta E \approx \sqrt{E_{beam} < E_{photon} >}$$

  and

$$\varepsilon_c[keV] = 2.218 \frac{E[GeV]^3}{\rho[m]} = 0.665 \cdot E[GeV]^2 \cdot B[T]$$

For our Higgs factory ε_{crit} = 1.27 MeV

- Therefore

$$\frac{\Delta E}{E} \approx \frac{\sigma_p}{p} \approx 0.0033$$

- The rf-bucket must contain this energy spread in the beam
- As U_o ~ 6.2 GeV,

  $V_{rf,max}$ > 6.2 GeV + "safety margin" to contain ΔE/E
- Some addition analysis

$$\left(\frac{\Delta E}{E}\right)_{\max} = \sqrt{\frac{q\hat{V}_{\max}}{\pi h \alpha_c E_{sync}} \left(2\cos\varphi_s + (2\varphi_s - \pi)\sin\varphi_s\right.}$$

- The greater the over-voltage, the shorter the bunch

$$\sigma_S = \frac{c\eta_C}{\Omega}\frac{\sigma_p}{p_0} = \sqrt{\frac{c^3}{2\pi q}\frac{p_0\beta_0\eta_C}{h f_0^2 \hat{V}_{\max}\cos(\varphi_S)}}\frac{\sigma_p}{p_0}$$

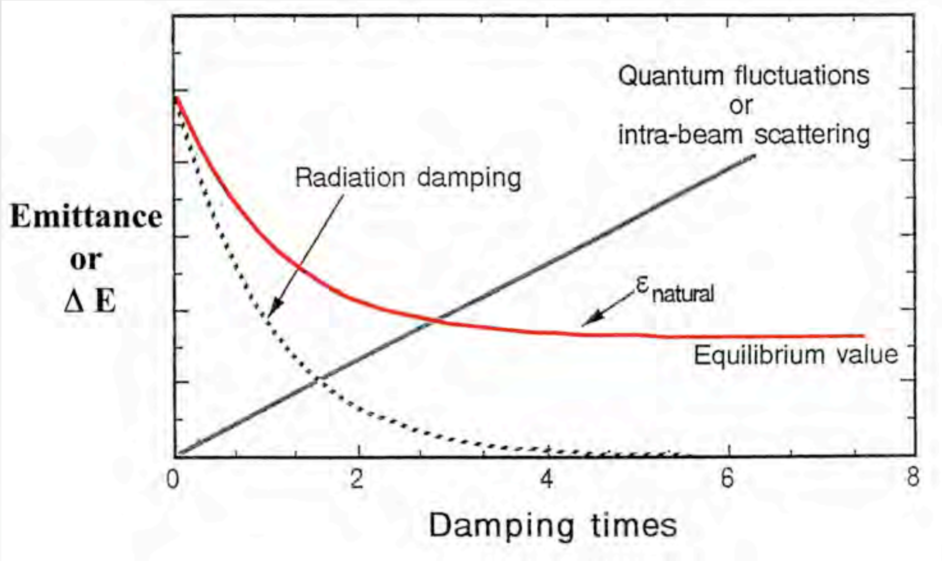
For the Higgs factory…

- The maximum accelerating voltage must exceed 9 GeV
  - Also yields σ_z = 3 mm which is okay for ß* = 1 mm
- A more comfortable choice is 11 GeV (it's only money)
  - ==> CW superconducting linac for LEP 3
  - This sets the synchronous phase
- For the next step we need to know the beam size

$$\sigma_i^* = \sqrt{\beta_i^* \varepsilon_i} \quad \text{for } i = x, y$$

- Therefore, we must estimate the natural emittance which is determined by the synchrotron radiation ΔE/E

The minimum horizontal emittance for an achromatic transport



$$\varepsilon_{x,\min} = 3.84 \times 10^{-13} \left(\frac{\gamma^2}{J_x} \right) F^{\min} \text{ meters}$$

$$\approx 3.84 \times 10^{-13} \gamma^2 \left(\frac{\theta^3_{achromat}}{4\sqrt{15}} \right) \text{ meters}$$

$$\varepsilon_y \sim 0.01\ \varepsilon_x$$

Because α_c is so small, we cannot achieve the minimum emittance

- For estimation purposes we will choose 20 ε_{min} as the mean of the x & y emittances
- For the LHC tunnel a maximum practical ipole length is 15 m
  - ➙ A triple bend achromat ~ 80 meters long ==> $\theta = 2.67 \times 10^{-2}$

$$\langle \varepsilon \rangle \sim 7.6 \text{ nm-rad} \Longrightarrow \sigma_{transverse} = 2.8\ \mu\text{m}$$

How many particles are in the bunch?

Or how many bunches are in the ring?

We already assumed that the luminosity is at the tune-shift limit

- We have

$$L = \frac{N^2 c\gamma}{4\pi\varepsilon_n \beta^* S_B} = \frac{1}{e r_i m_i c^2}\frac{N r_i}{4\pi\varepsilon_n}\left(\frac{EI}{\beta^*}\right) = \frac{1}{e r_i m_i c^2}\frac{N r_i}{4\pi\varepsilon_n}\left(\frac{P_{beam}}{\beta^*}\right) \qquad i = e, p$$

Linear or Circular

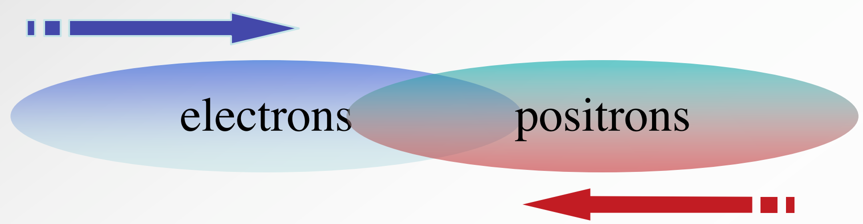
Tune shift

- Or $$Q = \frac{N r_e}{4\pi\varepsilon\gamma} \Rightarrow N = \frac{4\pi\varepsilon\gamma}{r_e} Q$$

- So, $N_e \sim 8 \times 10^{11}$ per bunch

- I_{beam} = 7.5 mA ==> there are only 5 bunches in the ring

Space charge fields at the collision point



At Interaction Point space charge cancels; currents add

==> strong beam-beam focus

--> Luminosity enhancement

--> Very strong synchrotron radiation

This is important in linear colliders

What about the beams in LEP-3?

At the collision point…

$$I_{peak} = N_e / 4\,\sigma_z \implies I_{peak} = 100\ \text{kA}$$

- Therefore, at the beam edge (2σ)

$$B = I(A)/5r(cm) = 36\ \text{MG !}$$

- When the beams collide they will emit synchrotron radiation (beamstrahlung)

$$\varepsilon_c[keV] = 2.218 \frac{E[GeV]^3}{\rho[m]} = 0.665 \cdot E[GeV]^2 \cdot B[T]$$

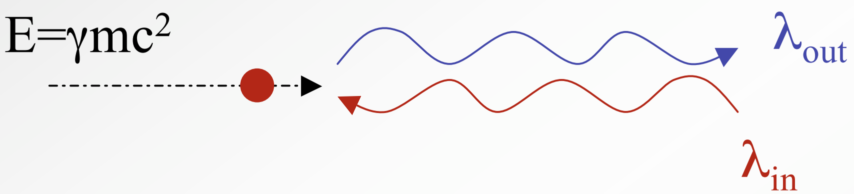
- For LEP-3 E_{crit} = 35 GeV ! (There are quantum corrections)

The rf-bucket cannot contain such a big ΔE/E

Beamstrahlung limits beam lifetime & energy resolution of events

There are other problems

- Remember the Compton scattering of photons up shifts the energy by $4\gamma^2$



- Where are the photons?
  - The beam tube is filled with thermal photons (25 meV)

- In LEP-3 these photons can be up-shifted as much as 2.4 GeV
  - 2% of beam energy cannot be contained
  - We need to put in the Compton cross-section and photon density to find out how rapidly beam is lost